

Stringent bounds on the brane width from stellar interferometry and distant gamma ray bursts: Back to the hierarchy problem?

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A simple idea restricting the brane width due to astronomical observations is proposed. Not to contradict the observational data the brane width should be of about Planck size giving therefore strict criterion in selecting the realistic braneworld models.

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If the background metric undergoes quantum fluctuations, the fluctuations will show up in measuring the space-time intervals. Therefore, if it is principally impossible to measure the space-time intervals precisely, this intrinsic limitation is naturally interpreted as a result of background metric fluctuations. Hence, the importance of carrying out the space-time measurement to evaluate the fluctuations of the background metric cannot be over-emphasized. To measure the distance between two points we need the clock and the mirror situated at those points respectively. The measurement is performed by sending the light signal from clock to the mirror where it is reflected and returns to the clock. However, quantum fluctuations in the positions of the clock and mirror introduce an inaccuracy in distance measurement [1]. Uncertainties contributed by the clock and the mirror to the measurement can be accurately evaluated [2]. In the case of brane there is an additional source of error in length measurement on the brane introduced by the brane width. Usually, the standard model fields are confined to the brane within some localization width, i.e., brane width [3, 4]. If the brane width is ϵ it means that brane localized particle probes this length scale across the brane and therefore the observer can not measure the distance on the brane to a better accuracy than ϵ . This result immediately indicates that brane localized observer can never know a time duration to a better accuracy than ϵ , (we are using the system of units $\hbar = c = 1$). The uncertainties in space-time measurements naturally produce the uncertainties in energy-momentum measurements, for the particle with momentum p has the wavelength $\lambda = 2\pi p^{-1}$ and due to length uncertainty one finds $\delta p = 2\pi\lambda^{-2}\delta\lambda$, $\delta E = pE^{-1}\delta p$. So that the energy-momentum uncertainties of a brane localized particle caused by the brane width read

$$\delta p = \frac{p^2\epsilon}{2\pi}, \quad \delta E = \frac{(E^2 - m^2)^{3/2}\epsilon}{2\pi E}. \quad (1)$$

From Eq.(1) one sees that the energy scale E for the particle with mass $m \ll E$ and the localization width ϵ is defined with accuracy $\delta E \approx E^2\epsilon/2\pi$. In the case $\epsilon \sim \text{TeV}^{-1}$ as it is usually assumed in braneworld models with large extra dimensions [4], for ultra high energy cosmic rays, $E \sim 10^8 \text{TeV}$, one gets $\delta E \sim 10^{16} \text{TeV}$. Thus, the very

high energy cosmic rays place the bound $\delta E \lesssim 10^8 \text{TeV}$, which implies

$$\epsilon \lesssim 3.894 \times 10^{-24} \text{cm}.$$

The energy-momentum uncertainties due to brane width result in uncertainties of phase and group velocities of the photon

$$\delta v_p = \frac{\delta E}{p} + \frac{E\delta p}{p^2} = \frac{E\epsilon}{\pi}, \quad \delta v_g = \frac{d\delta E}{dp} = \frac{E\epsilon}{\pi}. \quad (2)$$

But is such an effect observable? Let us follow an interesting idea proposed in [5]. The idea is to consider the phase incoherence of light coming to us from extragalactic sources. Since the phase coherence of light from an astronomical source incident upon a two-element interferometer is necessary condition to subsequently form interference fringes, such observations offer by far the most sensitive and uncontroversial test. The light with wavelength λ traveling over a distance l accumulates the phase uncertainty

$$\delta\varphi = \frac{2\pi l}{\lambda} \delta \left(\frac{v_p}{v_g} \right) = \frac{4lE\epsilon}{\lambda}.$$

The interference pattern when the source is viewed through a telescope will be destroyed if $\delta\varphi$ approaches 2π . In other words, if the light with wavelength λ received from a celestial optical source located at a distance l away produces the normal interference pattern, the corresponding phase uncertainty should satisfy the condition

$$\delta\varphi = \frac{8\pi l\epsilon}{\lambda^2} < 2\pi. \quad (3)$$

Consider the examples used in [5]. The Young's type of interference effects were clearly seen at $\lambda = 2.2\mu\text{m}$ light from a source at 1.012 kpc distance, viz. the star S Ser, using the Infra-red Optical Telescope Array, which enabled a radius determination of the star [6]. From Eq.(3) one gets

$$\epsilon \lesssim 0.385 \times 10^{-29} \text{cm}.$$

Airy rings (circular diffraction) were clearly visible at both the zeroth and first maxima in an observation of

the active galaxy PKS1413+135 ($l = 1.216$ Gpc) by the Hubble Space Telescope at $1.6\mu\text{m}$ wavelength [7]. Correspondingly, using the Eq.(3) one gets the following restriction on epsilon

$$\epsilon \lesssim 0.169 \times 10^{-35} \text{cm} .$$

The above consideration tells us that the error made in measuring the length l by the light with wave length λ approaches

$$\delta l = \epsilon \frac{l}{\lambda} ,$$

since the wavelength can not be known with a better accuracy than ϵ . The uncertainty in length measurement will lead to apparent blurring of distant point sources observed through the telescope [8]. Considering the distances l_1 and l_2 as measured from a point source placed at a distance $l \approx l_1 \approx l_2$ from the two sides of a telescope of aperture D , any intrinsic variation δl in the wavefront along the two lines of sight will translate into an apparent angular shift $\delta\theta$ given by [8]

$$\delta\theta \approx \frac{\delta l}{D} . \quad (4)$$

So one can state that the fluctuations in space-time measurements on the brane due to brane width lead to an apparent angular broadening of a light source placed at a distance l , as seen from a telescope of diameter D , given by

$$\delta\theta \approx \frac{\epsilon l}{\lambda D} . \quad (5)$$

From this point of view, the analyses of diffraction pattern from the Hubble Space Telescope observations of SN 1994D [9] and Hubble Deep Field high- z images [10] puts the following restriction on the brane width [8]

$$\epsilon \lesssim 3 \times 10^{-36} \text{cm} .$$

Another idea is to use experimental limits on the variation of light speed. From Eq.(2) one sees that fluctuations of light speed increase with energy. Thus for photons emitted simultaneously from a distant source coming towards our detector, we expect an energy dependent spread in their arrival times. To maximize the spread in arrival times, it is desirable to look for energetic photons from distant sources. This proposal was first made in another context in [11]. The analyses of the most rapid TeV flare observed thus far from active galaxy Markarian 421 on 15 May 1996 [12] puts the following limit on the brane width [13, 14]

$$\epsilon \lesssim \frac{\pi}{4} \times 10^{-16} \text{GeV}^{-1} \approx 0.973 \times 10^{-29} \text{cm} .$$

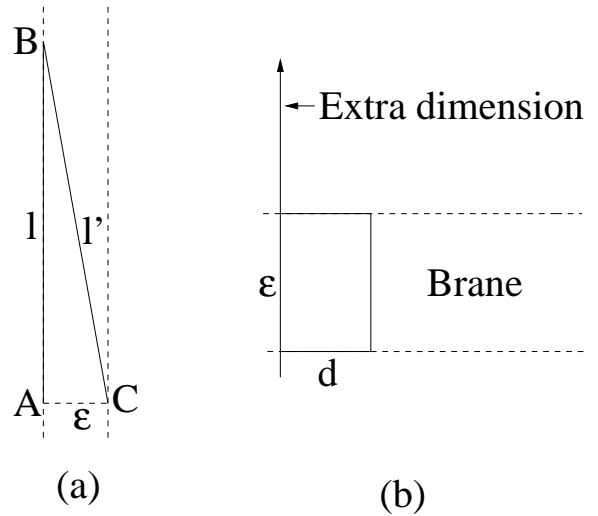


FIG. 1: (a) The light signal traveling the path clock-mirror-clock; (b) “Distortion” of the clock due to brane width.

At the first glance one can think the situation is not so “hopeless” as it is depicted above. Namely one can consider a schematic picture of the measurement given in Fig.1 (a). Clock sending the light from point A to the mirror situated at distance l (point B) can detect the signal coming back at point C (for the clock undergoes quantum fluctuations across the brane) and therefore uncertainty in length measurement should be $l' - l \sim \epsilon^2/l$, which is much less than ϵ for relatively large distances $l \gg \epsilon$. Unfortunately, clock can not measure the time with accuracy greater than ϵ . Imagine the clock as a mirror box with linear size d inside which light is bouncing. In absence of extra dimensions the resolution time of this clock will be d . But in the case of braneworld scenario, the light is localized within ϵ along the extra dimension(s) and therefore our clock is a box $\epsilon \times d$ inside which light is bouncing, Fig.1 (b). Hence, the accuracy of the clock can not be greater than ϵ , no matter what the length scale d is. During the time interval ϵ the clock is capable to resolve, the light will travel a distance ϵ resulting thereby this error in the length measurement. So, one sees that the brane width actually represents an error in length measurement on the brane.

Strictly speaking the experimental results considered here have to do mainly with the photon localization width, which in general may be different from the localization widths of other species of particles. However, if we want to address the hierarchy problem they ought not to differ very much. But even in this case we need to explain the natural appearance of so small brane width from higher-dimensional theory with TeV fundamental scale. Without this attempt we are left with the hierarchy (problem?) between the size of extra dimensions $\sim \text{TeV}^{-1}$ and the brane width that should be of about Planck size.

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